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The Effect of the Transparency of Order Flows in a Dealer Market with Several Securities

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This paper compares two trading mechanisms in a dealer market with several securities exhibiting asymmetric information and imperfect competition. These two market structures differ in the information received by market-makers. While in the first of them when setting the price of an asset, they observe the order flows of all assets, in the second one they only observe the order flow corresponding to this asset. In order to make this comparison, we analyze several market indicators such as the informed expected traded volume, the market depth, the volatility and the informativeness of equilibrium prices, and the informed traders' ex-ante expected profits. *Journal of Economic Literature* Classification Number: G10.

1. INTRODUCTION

One of the most surprising phenomena in asset markets over the past decade has been the proliferation of new markets. This growth leads to the competition between them and, hence, it is natural to wonder which market designs will survive.

In this paper, we turn our attention to a specific issue in market performance, which is transparency. O'Hara (1995) defines it as the ability of market participants to observe the information in the trading process. She emphasizes that, despite the simplicity of this definition, this issue is complex. One difficulty is to make concrete what information is observable. A second difficulty is to determine who can observe the information.

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Building on this insight, we compare two kinds of market structures, referred to as centralized and fragmented markets. In a centralized market, the orders of all the securities are addressed to the same location. Consequently, we can interpret this structure as a unique market for all the securities. On the other hand, in a fragmented market, orders of different securities are addressed to different locations. So, we can interpret that there is one market for each financial asset. The difference between these two market structures is the information available to agents who provide liquidity.

To perform this comparison, we contrast some properties of two trading mechanisms in the context of a static, imperfectly competitive, multiasset market. More precisely, we compare two distinct multisecurity extensions of the Kyle's (1985) framework proposed by Caballé and Krishnan (1994) and Bossaerts (1993). Whereas in the first work, market-makers observe the order flows of all the assets when they set prices, in the second work, market-makers can only observe the order flow of the security that they price. Notice that Caballé and Krishnan present a centralized financial market, while Bossaerts models a fragmented market.

It is clear that, on the one hand, the difference between these two market structures affects the behavior of market-makers since the prices they post depend on their information set. On the other hand, informed traders realize this difference and, hence, their strategies differ in both regimes. Consequently, the equilibria obtained in Caballé and Krishnan (1994) and Bossaerts (1993), in general, are distinct. It would therefore be interesting, from theoretical as well as policy standpoints, to examine the different aspects in which centralized and fragmented markets differ. This motivates the five questions I address in this paper:

1. How does informed trading differ under the two market structures?
2. How does market depth differ?
3. How do the two market structures differ in terms of price volatility?
4. In which market structure is the equilibrium price structure more informative?
5. Which market structure do informed traders earn a higher expected profit in?

My analysis produces the following answers. First, we find that informed traders expect to trade the same under both market structures. In relation to the market depth, we obtain an ambiguous result. Concerning the volatility of prices, we show that the variances of the equilibrium prices of each security under both regimes are equal. In relation to the informational content of prices, we prove that the equilibrium price vector corresponding to the fragmented trading mechanism is more informative about the payoff of an asset than the corresponding to the centralized mechanism. With regard to the informed traders' ex-ante expected profits, we show that they are smaller in the centralized trading mechanism than in the fragmented mechanism.

Previous studies have examined how differences in information about the market itself influence the performance of the market. Madhavan (1992) analyzes how transparency affects market behavior and viability. Biais (1993) examines how transparency of quotes affects spreads when there is no private information. Pagano and Röell (1996) compare the price formation in several trading systems, which differ in the degree of transparency. They analyze how transparency affects the trading costs for uninformed traders. Other papers (see, for instance, Röell (1990), Admati and Pfleiderer (1991), and Fishman and Longstaff (1992)) focus on the notion of transparency referred to the degree to which agents can trade anonymously. Our concept of transparency is related to market orders rather than to identities of agents. The present paper differs from all the previous papers in the fact that we examine a multiasset framework. In this richer context, a new notion of transparency and its economic implications can be analyzed.

The remainder of this paper is organized as follows. Section 2 outlines the notation and the hypotheses, which are common for both settings. Section 3 and Section 4 state the unique linear equilibrium corresponding to the centralized and the fragmented market, respectively. In Section 5 we compare some indicators of market performance. Concluding comments are presented in Section 6. Finally, all the proofs are included in the Appendix.

2. THE COMMON FRAMEWORK

We establish the following notation: if D is a $N \times N$ matrix, then $(D)_n$ will be its nth row and $(D)_{n,n'}$ will denote its (n, n') element, for any $n, n' \in \{1, \ldots, N\}$. If D is a symmetric positive definite matrix, then $D^{1/2}$ will mean the unique symmetric positive definite square root of D.[2] The superscript T either on a vector or on a matrix will denote its transpose.

Consider a financial market with N securities. Let $\tilde{v}$ be the payoff vector, $\tilde{v} = (\tilde{v}_1, \ldots, \tilde{v}_N)^T$, which has a multivariate normally *(MN)* distribution with mean vector $\bar{v}$ and a nonsingular variance matrix Σ_v. Three kinds of agents participate in the market: noise traders, informed investors, and market-makers. Noise traders demand a vector of random, inelastic quantities, not based on maximizing behavior, denoted by $\tilde{z} = (\tilde{z}_1, \ldots, \tilde{z}_N)^T$, which is $MN(\bar{z}, \Sigma_z)$, where Σ_z is nonsingular.

There are K risk-neutral informed investors, indexed by k. These agents possess private information about the payoff vector. Let $\tilde{s}_k$ represent the vector of signals received by informed trader k and let $\tilde{\xi}_k$ denote his informational advantage, defined as $\tilde{\xi}_k = E(\tilde{v} \mid \tilde{s}_k) - \bar{v}$, which is $MN(\vec{0}, \Sigma_\xi)$, where $\vec{0}$ denotes the zero

[2] $D^{1/2} = E\Lambda^{1/2}E^T$, where $\Lambda^{1/2}$ is a diagonal matrix with the positive square roots of the eigenvalues of D in the diagonal, and E is an orthogonal matrix, whose columns are eigenvectors of D (see Bellman (1970)).

vector and the variance matrix Σ_ξ is nonsingular.[3] In addition, we assume that for any $k, j \in \{1, \ldots, K\}$, with $k \neq j$, $\mathrm{cov}(\tilde{\xi}_k, \tilde{\xi}_j) = \Sigma_c$, where Σ_c is symmetric and positive definite. The random vectors $\tilde{v}, \tilde{\xi}_1, \ldots, \tilde{\xi}_K$ are assumed to be independent of $\tilde{z}$.

Market-makers are risk-neutral and set the prices of the securities they trade, after observing their information set. We assume competition among market-makers, so that this forces them to choose prices such that they earn zero expected profits.

3. THE CENTRALIZED MECHANISM

The framework of this section is the one proposed by Caballé and Krishnan (1994). Let $\tilde{x}_k = x_k(\tilde{\xi}_k)$ be the demand of informed trader k, which is a N-dimensional random vector. In this setup we suppose that market-makers are able to observe the order flows of all the assets. Thus, the zero expected profits condition implies that the price vector $\tilde{p}$ satisfies

$$\tilde{p} = p(\tilde{\omega}) = E(\tilde{v} \mid \tilde{\omega}), \quad \text{a.s.,} \tag{1}$$

where $\tilde{\omega} = \sum_{k=1}^{K} \tilde{x}_k + \tilde{z}$ is the vector of order flows.

DEFINITION 1. An equilibrium is a vector of strategies $x = ((x_1(\tilde{\xi}_1))^T, \ldots, (x_K(\tilde{\xi}_K))^T)^T$ and a price vector $\tilde{p} = p(\tilde{\omega})$ such that

1. for any $k \in \{1, \ldots, K\}$ and for any alternative vector of strategies x' differing from x only in the kth component, it holds that

$$E\left[\left(\tilde{v} - p\left(\sum_{j=1}^{K} x_j(\tilde{\xi}_j) + \tilde{z}\right)\right)^T x_k(\tilde{\xi}_k)\right]$$
$$\geq E\left[\left(\tilde{v} - p\left(\sum_{j \neq k} x_j(\tilde{\xi}_j) + x'_k(\tilde{\xi}_k) + \tilde{z}\right)\right)^T x'_k(\tilde{\xi}_k)\right], \quad \text{and}$$

2. $\tilde{p}$ satisfies (1).

For the sake of tractability, we will focus on linear equilibria. Caballé and Krishnan (1994) provide the explicit characterization of the unique linear equilibrium in this setup, which is stated in the following result:

[3] As in Caballé and Krishnan (1994), and in contrast to Bossaerts (1993), instead of using the signals received by informed traders, we perform the analysis with the informational advantages, since in this way we allow the dimension of the space of signals not to coincide with the number of securities of the market.

PROPOSITION 1. *There exists a unique linear equilibrium defined as*

$$p(\tilde{\omega}) = \bar{v} + \frac{\sqrt{K}}{2}A(\tilde{\omega} - \bar{z}) \quad \textit{and} \quad x_k(\tilde{\xi}_k) = B\tilde{\xi}_k, \quad \textit{for all } k,$$

where

$$A = \Sigma_z^{-1/2} M^{1/2} \Sigma_z^{-1/2} \quad \textit{and} \quad B = \frac{1}{\sqrt{K}} A^{-1}\left(I + \frac{(K-1)}{2}\Sigma_c \Sigma_\xi^{-1}\right)^{-1}, \quad \textit{with}$$

$$M = \Sigma_z^{1/2} G \Sigma_z^{1/2} \quad \textit{and}$$

$$G = \left(I + \frac{(K-1)}{2}\Sigma_c \Sigma_\xi^{-1}\right)^{-1} \Sigma_\xi \left(I + \frac{(K-1)}{2}\Sigma_\xi^{-1}\Sigma_c\right)^{-1}. \tag{2}$$

Notice that the equilibrium coefficient matrix A governing the relationship between the vector of prices and the vector of order flows is symmetric and positive definite. This is a consequence of strategic behavior since informed investors trade more aggressively in the markets where there is more camouflage. Thus the nth price responds to the qth order flow exactly as the qth price responds to the nth order flow, for all pair of assets n and q (see Caballé and Krishnan (1994) for a further discussion of this property).

4. THE FRAGMENTED MECHANISM

The setting of this section is related to the one chosen by Bossaerts (1993). The superscript F refers to the fragmented mechanism. In this framework market-makers' quotes can only depend on the order flow of their own market. Using the zero expected profits condition, we obtain that, for any $n \in \{1, \ldots, N\}$, the selected price of the nth security satisfies

$$\tilde{p}_n^F = p_n^F\left(\tilde{\omega}_n^F\right) = E\left(\tilde{v}_n \mid \tilde{\omega}_n^F\right), \quad \text{a.s.}, \tag{3}$$

where $\tilde{\omega}_n^F = \sum_{k=1}^K \tilde{x}_{k,n}^F + \tilde{z}_n$ is the order flow corresponding to the nth security.

DEFINITION 2. An equilibrium is a vector of strategies $x^F = ((x_1^F(\tilde{\xi}_1))^T, \ldots, (x_K^F(\tilde{\xi}_K))^T)^T$ and a price vector $\tilde{p}^F = p^F(\tilde{\omega}^F)$ such that

1. for any $k \in \{1, \ldots, K\}$ and for any alternative vector of strategies x'^F differing from x^F only in the kth component, it holds that

$$E\left[\sum_{n=1}^N \left(\tilde{v}_n - p_n^F\left(\sum_{j=1}^K x_{j,n}^F(\tilde{\xi}_j) + \tilde{z}_n\right)\right) x_{k,n}^F(\tilde{\xi}_k)\right]$$
$$\geq E\left[\sum_{n=1}^N \left(\tilde{v}_n - p_n^F\left(\sum_{j\neq k} x_{j,n}^F(\tilde{\xi}_j) + x_{k,n}'^F(\tilde{\xi}_k) + \tilde{z}_n\right)\right) x_{k,n}'^F(\tilde{\xi}_k)\right], \quad \text{and}$$

2. for all $n \in \{1, \ldots, N\}$, the nth component of $\tilde{p}^F$ satisfies (3).

Remark. Notice that this definition coincides with Definition 1, except in the fact that now the price of an asset is contingent only in its own order flow.

The following result finds the unique linear equilibrium in closed form.

PROPOSITION 2. *There exists a unique linear equilibrium defined as*

$$p^F(\tilde{\omega}^F) = \bar{v} + \frac{\sqrt{K}}{2} A^F(\tilde{\omega}^F - \bar{z}) \quad \textit{and} \quad x_k^F(\tilde{\xi}_k) = B^F \tilde{\xi}_k, \quad \textit{for all } k,$$

where A^F is a $N \times N$ diagonal matrix, with

$$(A^F)_{n,n} = \sqrt{\frac{(G)_{n,n}}{(\Sigma_z)_{n,n}}}, \quad \textit{for all } n, \textit{ and} \quad B^F = \frac{1}{\sqrt{K}}(A^F)^{-1}\left(I + \frac{(K-1)}{2}\Sigma_c \Sigma_{\xi}^{-1}\right)^{-1}.$$

It is important to point out that only the diagonal of Σ_z is relevant in the equilibrium coefficients that we have just derived. This property follows from the fact that market-makers cannot take into account the interactions between different components of the vector of demands of noise traders.

5. COMPARISON

In this section we examine the differences between the centralized and the fragmented market in terms of some indicators of market performance.

The Informed Expected Trading Volume

COROLLARY 1. *For any $n \in \{1, \ldots, N\}$, the expected trading volume for any informed trader in asset n is the same in both setups.*

This result tells us that in equilibrium informed traders expect to take the same positions in both market structures for any asset. Nevertheless, as we see later on, this does not mean that informed traders expect the same profits in both setups.

The Market Depth

To measure the depth for asset n—the order flow corresponding to this asset necessary to induce the price of asset n to rise or fall by one unity—we should compare the inverse of the nth diagonal element of the matrices A and A^F. The results obtained from the comparison of the market depth are ambiguous in the sense that, in general, we cannot determine which trading mechanism leads to a deeper market. In order to illustrate this, we present the following two examples based on the simple framework where there are two assets ($N = 2$) with $\bar{v} = \vec{0}$, and a single informed trader ($K = 1$) with perfect information ($\tilde{\xi}_1 = \tilde{v}$).

EXAMPLE 1.

$$\Sigma_\xi = \Sigma_v = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}, \quad \Sigma_z = \begin{bmatrix} 1 & \rho \\ \rho & 1 \end{bmatrix}, \qquad \text{with } 0 < \rho < 1.$$

Using Propositions 1 and 2, we know that in this case $A = \Sigma_z^{-1/2}$ and $A^F = I$. Hence, we have that $A_{n,n} < (A^F)_{n,n}$. In order to understand these inequalities and, without loss of generality, let us focus on asset 1. Combining (1), (3), and the expressions of equilibrium prices stated in Propositions 1 and 2, it is easy to see that

$$A_{1,1} = 2\frac{\partial E(\tilde{v}_1 \mid \tilde{\omega}_1, \tilde{\omega}_2)}{\partial \tilde{\omega}_1} \quad \text{and} \quad A_{1,1}^F = 2\frac{\partial E\left(\tilde{v}_1 \mid \tilde{\omega}_1^F\right)}{\partial \tilde{\omega}_1^F}.$$

Therefore, it suffices to explain how market-makers interpret an increase in the realization of order flow of asset 1, remaining fixed the realization of order flow of asset 2. In the centralized market a change in $\tilde{\omega}_1$ provides both a direct and an indirect information about $\tilde{v}_1$, which are opposite. Note that, since $\tilde{\omega}_1 = \frac{1}{\det(A)}(A_{2,2}\tilde{v}_1 - A_{1,2}\tilde{v}_2) + \tilde{z}_1$, the payoffs are uncorrelated and A is a positive definite matrix, an increase in the realization of $\tilde{\omega}_1$ translates into a higher expected value of $\tilde{v}_1$. This is the direct effect. However, $\tilde{\omega}_1$ is also informative about $\tilde{\omega}_2$, where $\tilde{\omega}_2 = \frac{1}{\det(A)}(-A_{1,2}\tilde{v}_1 + A_{1,1}\tilde{v}_2) + \tilde{z}_2$. Moreover, since the payoffs are uncorrelated, the noise demands are positively correlated and $A_{1,2} < 0$, a higher value of $\tilde{\omega}_1$ causes an increase in the expected value of

$$\frac{A_{1,1}\tilde{v}_2}{\det(A)} + \tilde{z}_2.$$

But since $\tilde{\omega}_2$ is fixed, this leads to a decrease in the expected value of $(-A_{1,2}\tilde{v}_1)/(\det(A))$, which is equivalent to a decrease in the expected $\tilde{v}_1$ because $A_{1,2} < 0$. This is the indirect effect. By contrast, in the fragmented market, a change in $\tilde{\omega}_1^F$ only generates the direct effect.[4] In short, in the centralized market there is an additional effect (the indirect effect) that partially offsets the direct effect. This is the reason why in this example the market-makers' response to a change in the order flow of asset 1 is weaker in the centralized market.

EXAMPLE 2.

$$\Sigma_\xi = \Sigma_v = \begin{bmatrix} 1 & \rho \\ \rho & 1 \end{bmatrix}, \qquad \text{with } 0 < \rho < 1,$$

[4] The indirect effect does not have sense because in this case the market-makers' information set consists of a single order flow.

and

$$\Sigma_z = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}.$$

Using Propositions 1 and 2, we get that $A = \Sigma_\xi^{1/2}$ and $A^F = I$. Hence, it follows that $A_{n,n} > (A^F)_{n,n}$, for all n. Again, in order to understand intuitively these inequalities, let us turn our attention to asset 1. Now, in the centralized market the direct and the indirect effect have the same sign. As before, $\tilde{\omega}_1$ and $\tilde{v}_1$ are positively correlated and, consequently, the direct effect has positive sign. Concerning the indirect effect notice that, since the noise demands are uncorrelated and $A_{22}\rho - A_{1,2} > 0$, a higher value of $\tilde{\omega}_1$ causes an increase in the expected value of

$$\frac{A_{1,1}\tilde{v}_2}{\det(A)} + \tilde{z}_2.$$

But since $\tilde{\omega}_2$ remains fixed, this leads to a decrease in the expected value of $(-A_{1,2}\tilde{v}_1)/(\det(A))$, which is equivalent to a rise in the expected value of $\tilde{v}_1$ because $A_{1,2} > 0$. Therefore, in the centralized market there is an additional effect (the indirect effect) that has the same sign as the direct effect. This makes the market-makers' response to a change in the order flow of asset 1 be stronger in the centralized market.

The Volatility of Prices

Before comparing the volatility of prices, the following result provides some properties of equilibrium prices, which will facilitate the computation of the mentioned market indicator.

COROLLARY 2. *In equibrium, the following are true*:

(a) $\text{var}(\tilde{p}) = \text{cov}(\tilde{v}, \tilde{p})$,
(b) *for any* $n \in \{1, \ldots, N\}$, $\text{var}(\tilde{p}_n^F) = \text{cov}(\tilde{v}_n, \tilde{p}_n^F)$, *and*
(c) $\text{cov}(\tilde{v}, \tilde{p}) = \text{cov}(\tilde{v}, \tilde{p}^F)$.

As shown in the proof, the first two parts of Corollary 2 directly follow from the Projection Theorem. For part (c), notice that the equilibrium price vectors only differ in a term which only depends on noise demands, which are uncorrelated with the payoff vector (see Propositions 1 and 2).

The volatility of an individual price is measured by its variance. Notice that, from Corollary 2, it directly follows that the volatility of each price is the same in both settings.

COROLLARY 3. *For any* $n \in \{1, \ldots, N\}$, $\text{var}(\tilde{p}_n) = \text{var}(\tilde{p}_n^F)$.

We define the volatility of the price vector as its variance matrix. From the expressions of equilibrium price vectors stated in Propositions 1 and 2, we get that

$$\operatorname{var}(\tilde{p}) = K\left(2\Sigma_\xi^{-1} + (K-1)\Sigma_\xi^{-1}\Sigma_c\Sigma_\xi^{-1}\right)^{-1} \quad \text{and}$$

$$\operatorname{var}(\tilde{p}^F) = \frac{K}{4}\left[\left(I + \frac{(K-1)}{2}\Sigma_c\Sigma_\xi^{-1}\right)^{-1}(\Sigma_\xi + (K-1)\Sigma_c) \times \left(I + \frac{(K-1)}{2}\Sigma_\xi^{-1}\Sigma_c\right)^{-1} + H\right],$$

where H is a $N \times N$ matrix with its element (n, n') given by

$$(H)_{n,n'} = \sqrt{\frac{(G)_{n,n}}{(\Sigma_z)_{n,n}}}(\Sigma_z)_{n,n'}\sqrt{\frac{(G)_{n',n'}}{(\Sigma_z)_{n',n'}}}, \quad \text{for any } n, n' \in \{1, \ldots, N\}.$$

From the previous corollary we know that the diagonals of $\operatorname{var}(\tilde{p})$ and $\operatorname{var}(\tilde{p}^F)$ coincide. However, in general we cannot ensure that these two matrices are identical. Note that the formula of $\operatorname{var}(\tilde{p})$ is unaffected by the variance of noise trading, while $\operatorname{var}(\tilde{p}^F)$ may depend on it. This difference stems from the fact that the fragmented regime prevents market-makers from considering the possible correlations between the components of the noise demand, while the centralized regime allows them to do it.

The Informativeness of Prices

We define the informativeness of an individual price about the payoff of an asset as the reduction in the prior variance of the payoff, after conditioning on the individual price. The next corollary says that the informational content of an individual price about the payoff of a security is the same in both market structures.

COROLLARY 4 For any $n, n' \in \{1, \ldots, N\}$, $(\Sigma_v)_{n',n'} - \operatorname{var}(\tilde{v}_{n'} \mid \tilde{p}_n) = (\Sigma_v)_{n',n'} - \operatorname{var}(\tilde{v}_{n'} \mid \tilde{p}_n^F)$.

Using the normality assumption, we have that the informativeness of an individual price about the payoff of a security only depends on the covariance between this price and this payoff and the variance of this price. Corollary 4 follows from this fact and Corollaries 2 and 3.

Next, we want to contrast the infomational content of the price vectors about the payoff of an asset. The next corollary shows that $\tilde{p}^F$ is at least as informative about the payoff of an asset as $\tilde{p}$. At first glance, this result is puzzling because, when setting a price, market-makers observe a greater number of order flows in the centralized market. Corollary 5 is due to the strategic behavior of informed traders. They realize that in the more transparent market structure they cannot act "so aggressively" since otherwise they would reveal too much private information.

COROLLARY 5. *For any* $n \in \{1, \ldots, N\}$, $(\Sigma_v)_{n,n} - \text{var}(\tilde{v}_n \mid \tilde{p}^F) \geq (\Sigma_v)_{n,n} - \text{var}(\tilde{v}_n \mid \tilde{p})$.

Since Corollary 4 demonstrates that the informativeness of individual prices are the same in both market structures, one might be tempted to conclude that the informativeness of price vectors also coincide. However, in general this is not true because the correlation between prices is different in both market structures. To illustrate this result, we examine the particular case of two assets ($N = 2$). In the centralized market $\tilde{p}_2$ does not provide any additional information about $\tilde{v}_1$ to the information obtained from $\tilde{p}_1$. This is due to the fact that in this setting market-makers have the same information when posting $\tilde{p}_1$ and $\tilde{p}_2$. Formally, we have

$$\text{var}(\tilde{v}_1 \mid \tilde{p}_1, \tilde{p}_2) = \text{var}(\tilde{v}_1 \mid \tilde{p}_1, \tilde{p}_2 - E(\tilde{p}_2 \mid \tilde{p}_1)).$$

But, as $\text{cov}(\tilde{p}_1, \tilde{p}_2 - E(\tilde{p}_2 \mid \tilde{p}_1)) = 0$ and $\text{cov}(\tilde{v}_1, \tilde{p}_2 - E(\tilde{p}_2 \mid \tilde{p}_1)) = 0$, this yields that

$$\text{var}(\tilde{v}_1 \mid \tilde{p}_1, \tilde{p}_2) = \text{var}(\tilde{v}_1 \mid \tilde{p}_1).$$

This allows us to conclude that in the centralized market the price vector is as informative about $\tilde{v}_1$ as $\tilde{p}_1$. By contrast, in the fragmented mechanism markets markers have different information when posting different prices. In general, this implies that $\tilde{p}_2^F$ provides additional information about $\tilde{v}_1$ to the information obtained from $\tilde{p}_1^F$. Therefore, the revelation of information is smaller or equal through $\tilde{p}_1^F$ than through $\tilde{p}^F$. Corollary 5 stems from the combination of these facts and Corollary 4.

To end this point, note that Corollary 5 shows that for some parameter values we have that the informativeness of the price vectors are the same in the two market structures, and for some others we have that the price vector is more informative in the fragmented market. Next, two numerical examples are included to illustrate these two possibilities. First, consider the particular case where there are two assets ($N = 2$), a single trader ($K = 1$) and that $\Sigma_\xi = \Sigma_v = \Sigma_z = I$. It is easy to see that in this case the equilibrium price vector is the same in the two market structures and, obviously, we have an equality in Corollary 5. Next, consider the same parameter values, except that now the matrix Σ_z is not diagonal. In particular, suppose that

$$\Sigma_z = \begin{bmatrix} 1 & \rho \\ \rho & 1 \end{bmatrix}, \qquad \text{with } |\rho| < 1.$$

From direct computations, it follows that the informativeness of the price vector about the payoff of asset n is $\frac{1}{2}$ in the centralized market, and $\frac{2}{4-\rho^2}$ in the fragmented market. Therefore, $(\Sigma_v)_{n,n} - \text{var}(\tilde{v}_n \mid \tilde{p}^F) > (\Sigma_v)_{n,n} - \text{var}(\tilde{v}_n \mid \tilde{p})$.

The Market Participants' Expected Profits

In relation to market-makers, notice that in both mechanisms the Bertrand competition between these risk-neutral agents forces them to expect zero profits. Moreover, since we are in a zero-sum game, the expected trading costs for uninformed traders coincide with minus the informed traders' expected profits. Therefore, it suffices to examine the informed traders' expected profits.

Let us denote by $E(\Pi)$ and $E(\Pi^F)$ the ex-ante expected profits for an arbitrary informed investor obtained in both market structures. The next result provides that informed traders are better off in the fragmented mechanism.

COROLLARY 6. $E(\Pi^F) \geq E(\Pi)$.

The economic intuition of Corollary 6 is the following. In the centralized market the fact that market-makers observe all order flows when pricing an asset reduces the possibility that informed traders have to hide behind noise traders. Consequently, in the centralized market, informed traders are not able to exploit so much their private information as in the fragmented regime and, as a result of that, they are worse in the more transparent market mechanism.

As an illustration, we analyze the simple case where there is a single informed trader ($K = 1$). In this case,

$$E(\Pi) = \frac{1}{2} \sum_{n=1}^{N} \operatorname{cov}(\tilde{v}_n, x_{1,n})$$

and

$$E(\Pi^F) = \frac{1}{2} \sum_{n=1}^{N} \operatorname{cov}\left(\tilde{v}_n, x_{1,n}^F\right).$$

Therefore, the informed trader's expected profits depend on the covariance between the informed trader's demands and the payoffs of the assets. For instance, in both parameter sets stated in Examples 1 and 2, for all n, $\operatorname{cov}(\tilde{v}_n, x_{1,n}) < \operatorname{cov}(\tilde{v}_n, x_{1,n}^F)$ holds. This particular framework helps us to make clear what means that "... in the centralized market informed traders are not able to exploit so much their private information as in the fragmented regime..." Although the informed trader expects to trade the same in both regimes (recall Corollary 1), the lack of transparency in the fragmented market allows him to trade more with regard to his private information.

Our results here may be contrasted with those of Pagano and Röell (1996). These authors also investigate the economic implications of transparency in a model with a single risky asset. They conclude that in a Kyle's (1985) framework, transparency is irrelevant for uninformed traders' expected losses. This occurs because informed trades are indistinguishable from uninformed trades in either the nontransparent or transparent market setting. By contrast, here the possibility that informed traders have to hide behind noise traders differs across market structures.

6. CONCLUSIONS

The objective of this paper has been the comparison between two trading mechanisms in a dealer market, based on two distinct multi-security extensions of the Kyle's (1985) framework proposed by Caballé and Krishnan (1994) and Bossaerts (1993). The main conclusion to which this analysis leads is that informed traders can use the lack of transparency of the fragmented mechanism to exploit their private information. On the one hand, this implies that informed traders prefer the fragmented mechanism. On the other hand, this makes the vector of order flows (or, equivalently, the vector of prices) corresponding to the fragmented trading mechanism be at least as informative about the payoff of an asset as the corresponding to the centralized one.

Several extensions are left for future research. In the present paper, the presence of noise traders makes impossible to perform a rigorous welfare comparison. Therefore, it would be interesting to develop this analysis in a framework where uninformed traders behave rationally. Another future plan is the incorporation of risk-averse market participants. This would allow us to examine the robustness of our results.

APPENDIX

Proof of Proposition 1. See Caballé and Krishnan (1994). ■

Proof of Proposition 2. Notice that since we consider linear equilibria, we can write

$$\tilde{p}^F = A_0^F + A_1^F \tilde{\omega}^S \quad \text{and} \quad \tilde{x}_k^F = C_k^F + B_k^F \tilde{\xi}_k, \quad \text{for all } k, \tag{A1}$$

where A_0^F and C_k^F are N-dimensional vectors and A_1^F and B_k^F are $N \times N$ matrices, such that A_1^F is a diagonal matrix.

First, we consider the informed traders' decisions. By virtue of (A1), the quantities chosen by the kth informed trader $\tilde{x}_k^F$ solves the following optimization problem:

$$\max_{\tilde{x}_k^F} E\left[\left(\tilde{v} - A_0^F - A_1^F\left(\sum_{j\neq k}(C_j^F + B_j^F \tilde{\xi}_j) + \tilde{x}_k^F + \tilde{z}\right)\right)^T \tilde{x}_k^F \mid \tilde{\xi}_k\right].$$

The first order condition of this problem is

$$E(\tilde{v} \mid \tilde{\xi}_k) - A_0^F - A_1^F \bar{z} - A_1^F \sum_{j\neq k} C_j^F - A_1^F \sum_{j\neq k} \left\lfloor B_j^F E(\tilde{\xi}_j \mid \tilde{\xi}_k)\right\rfloor = 2A_1^F \tilde{x}_k^F \tag{A2}$$

and its second order condition implies the positive semidefiniteness of A_1^F. Using the assumptions of the informational advantages of informed traders, we have that $E(\tilde{v} \mid \tilde{\xi}_k) = \tilde{\xi}_k + \bar{v}$ and $E(\tilde{\xi}_j \mid \tilde{\xi}_k) = \Sigma_c \Sigma_\xi^{-1} \tilde{\xi}_k$, for all $j \neq k$. Plugging the last two expressions into (A2), using the linearity assumption conformably with (A1) and operating, we get

$$A_1^F C_k^F = \bar{v} - A_0^F - A_1^F \bar{z} - A_1^F \sum_{h=1}^{K} C_h^F \tag{A3}$$

and

$$I = A_1^F \left(2B_k^F + \sum_{j \neq k} B_j^F \Sigma_c \Sigma_\xi^{-1} \right). \tag{A4}$$

In particular, observe that (A4) provides the nonsingularity of A_1^F.

Now, we will show that a linear equilibrium must be symmetric in the informed traders' strategies. Fix $k, k' \in \{1, \ldots, K\}$, with $k \neq k'$. Since (A3) and (A4) are satisfied for all $k \in \{1, \ldots, K\}$, we have

$$A_1^F C_{k'}^F = \bar{v} - A_0^F - A_1^F \bar{z} - A_1^F \sum_{h=1}^{K} C_h^F \tag{A3$'$}$$

and

$$I = A_1^F \left(2B_{k'}^F + \sum_{j \neq k'} B_j^F \Sigma_c \Sigma_\xi^{-1} \right). \tag{A4$'$}$$

Note that (A3), (A3′), and the nonsingularity of A_1^F imply that

$$C_k^F = C_{k'}^F. \tag{A5}$$

On the other hand, using (A4), (A4′), and the nonsingularity of A_1^F, we get

$$2B_k^F + \sum_{j \neq k} B_j^F \Sigma_c \Sigma_\xi^{-1} = 2B_{k'}^F + \sum_{j \neq k'} B_j^F \Sigma_c \Sigma_\xi^{-1}.$$

Simplifying this expression and rearranging the resulting equation, we have

$$\left(B_k^F - B_{k'}^F\right)\left(2I - \Sigma_c \Sigma_\xi^{-1}\right) = 0. \tag{A6}$$

However, it easy to see that var$(\tilde{\xi}_k - \tilde{\xi}_j) = 2(\Sigma_\xi - \Sigma_c)$, with $j \neq k$, which provides that $\Sigma_\xi - \Sigma_c$ is positive semidefinite. This implies that $2\Sigma_\xi - \Sigma_c$ is positive definite, and hence, we get that $2I - \Sigma_c \Sigma_\xi^{-1}$ is nonsingular. Consequently, from

(A6), it follows that $B_k^F = B_{k'}^F$. Since both (A5) and the last equality hold for any $k, k' \in \{1, \ldots, K\}$, with $k \neq k'$, (A1) can be expressed as

$$\tilde{p}^F = A_0^F + A_1^F \tilde{\omega}^F \quad \text{and} \quad \tilde{x}_k^F = C^F + B^F \tilde{\xi}_k, \quad \text{for all } k. \tag{A7}$$

Hence, (A3) and (A4) can be written as

$$(K+1) A_1^F C^F = \bar{v} - A_0^F - A_1^F \bar{z}, \tag{A8}$$

and $A_1^F B^F (2I + (K-1)\Sigma_c \Sigma_\xi^{-1}) = I$. Moreover, since A_1^F is nonsingular, we can solve the last equation for B^F as

$$(B^F)_n = \frac{1}{\left(A_1^F\right)_{n,n}} \left(\left(2I + (K-1)\Sigma_c \Sigma_\xi^{-1}\right)^{-1}\right)_n, \quad \text{for all } n. \tag{A9}$$

We next determine the formula of $\tilde{p}_n^F$, for any $n \in \{1, \ldots, N\}$. Notice that $\tilde{\omega}_n^F = K C_n^F + (B^F)_n \sum_{j=1}^K \tilde{\xi}_j + \tilde{z}_n$, which is normally distributed. Computing $E(\tilde{v}_n \mid \tilde{\omega}_n^F)$, substituting the resulting formula into (3) and equating coefficients according to (A7), we get

$$\left(A_1^F\right)_{n,n} = \frac{K(\Sigma_\xi)_n((B^F)_n)^T}{(B^F)_n(K\Sigma_\xi + K(K-1)\Sigma_c)((B^F)_n)^T + (\Sigma_z)_{n,n}} \tag{A10}$$

and

$$A_{0,n}^F = \bar{v}_n - \left(A_1^F\right)_{n,n}\left(K C_n^F + \bar{z}_n\right). \tag{A11}$$

It remains to solve the system of equations formed by (A8)–(A11). Plugging (A9) into (A10) and performing some algebra in the resulting equation, taking into account (2), we get that $(\Sigma_z)_{n,n}((A_1^F)_{n,n})^2 = \frac{K}{4}(G)_{n,n}$. The positive definiteness of Σ_z, G and A_1^F allows us to solve the last equality for $(A_1^F)_{n,n}$ as

$$\left(A_1^F\right)_{n,n} = \frac{\sqrt{K}}{2}\sqrt{\frac{(G)_{n,n}}{(\Sigma_z)_{n,n}}}. \tag{A12}$$

Combining (A8) and (A11), we obtain that $(A_1^F)_{n,n} C_n^F = 0$. Therefore, $C_n^F = 0$. Hence, (A11) provides that $A_{0,n}^F = \bar{v}_n - (A_1^F)_{n,n}\bar{z}_n$. Taking into account that the last two equalities are satisfied for any $n \in \{1, \ldots, N\}$, (A7) provides that $\tilde{p}^F = \bar{v} + A_1^F(\tilde{\omega}^F - \bar{z})$ and $\tilde{x}_k^F = B^F \tilde{\xi}_k$, for all k. Finally, using (A9), (A12), and the previous two formulas, we obtain the desired expressions. ■

Proof of Corollary 1. This is just an application of the fact that if $X \sim N(0, \sigma_X^2)$, then $E(|X|) = \sqrt{\frac{2}{\pi}\sigma_X^2}$. ■

Proof of Corollary 2. (a) It is easy to see that $\text{cov}(\tilde{v}, \tilde{p}) = \text{cov}(\tilde{v} - \tilde{p}, \tilde{p}) + \text{var}(\tilde{p})$. Hence, all we need to prove is that $\text{cov}(\tilde{v} - \tilde{p}, \tilde{p}) = 0$. However, notice that (1) and the Projection Theorem [see, for instance, Gourieroux and Monfort (1989)] provide the last equality.

(b) It is left since it is very similar to the proof of part (a).

(c) From Propositions 1 and 2, we get that the parts of $\tilde{p}$ and $\tilde{p}^F$ which are correlated with $\tilde{v}$ coincide. Therefore, the result is obtained. ■

Proof of Corollary 3. It immediately follows from Corollary 2. ■

Proof of Corollary 4. It is easy to see that the normality assumption and Corollaries 2 and 3 provide the equality given in the statement of this corollary. ■

Proof of Corollary 5. Fix $n \in \{1, \ldots, N\}$. From the normality assumption and Corollary 2.a, it is easy to see that $(\Sigma_v)_{n,n} - \text{var}(\tilde{v}_n \mid \tilde{p}) = \text{var}(\tilde{p}_n)$. Combining Corollaries 2.a and 4, we can rewrite $(\Sigma_v)_{n,n} - \text{var}(\tilde{v}_n \mid \tilde{p}) = (\Sigma_v)_{n,n} - \text{var}(\tilde{v}_n \mid \tilde{p}_n^F)$. Furthermore, since $\text{var}(\tilde{v}_n \mid \tilde{p}_n^F) \geq \text{var}(\tilde{v}_n \mid \tilde{p}^F)$, we obtain the desired expression. ■

Proof of Corollary 6. In order to show this result, it suffices to prove that $E[(\tilde{v} - \tilde{p}^F)^T \tilde{z}] \leq E[(\tilde{v} - \tilde{p})^T \tilde{z}]$ because we are in two zero-sum games and the market-makers' expected profits are zero. Notice that since $\tilde{v}$ and $\tilde{z}$ are independent and $E(\tilde{p}) = E(\tilde{p}^F) = \bar{v}$, the last inequality is equivalent to $-\sum_{n=1}^N \text{cov}(\tilde{p}_n^F, \tilde{z}_n) \leq -\sum_{n=1}^N \text{cov}(\tilde{p}_n, \tilde{z}_n)$, which will be shown by proving that $\text{cov}(\tilde{p}_n^F, \tilde{z}_n) \geq \text{cov}(\tilde{p}_n, \tilde{z}_n)$, for any $n \in \{1, \ldots, N\}$. Now, fix $n \in \{1, \ldots, N\}$. Note that, from the expression of $\tilde{p}^F$ given in Proposition 2 it follows that

$$\text{cov}\left(\tilde{p}_n^F, \tilde{z}_n\right) = \frac{\sqrt{K}}{2}(A^F)_{n,n}(\Sigma_z)_{n,n},$$

which can be expressed as

$$\text{cov}\left(\tilde{p}_n^F, \tilde{z}_n\right) = \frac{\sqrt{K}}{2}\sqrt{\text{var}((A^F)_n \tilde{z})}\sqrt{\text{var}(\tilde{z}_n)}. \tag{A13}$$

From Propositions 1 and 2, and Corollary 3, it follows that $\text{var}((A^F)_n \tilde{z}) = \text{var}((A)_n \tilde{z})$. Hence, (A13) provides that

$$\text{cov}\left(\tilde{p}_n^F, \tilde{z}_n\right) = \frac{\sqrt{K}}{2}\sqrt{\text{var}((A)_n \tilde{z})}\sqrt{\text{var}(\tilde{z}_n)}.$$

On the other hand, from Proposition 1, we have that

$$\text{cov}(\tilde{p}_n, \tilde{z}_n) = \frac{\sqrt{K}}{2}\text{cov}((A)_n \tilde{z}, \tilde{z}_n).$$

Finally, comparing the last two expressions, one concludes that $\text{cov}(\tilde{p}_n^F, \tilde{z}_n) \geq \text{cov}(\tilde{p}_n, \tilde{z}_n)$. ■

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